“Windfalls, the ‘Horizon,’ and Related Concepts in the Permanent-Income Hypothesis”
by Milton Friedman


In the analysis of consumer behavior, one question that arises is how a consumer unit reacts to an unanticipated change in circumstances. According to the permanent-income hypothesis, the answer depends on how the change affects the consumer unit’s evaluation of its longer-term income prospects, as summarized in its estimated permanent income. The effect on permanent income, in its turn, depends on the “horizon” of the consumer unit. A unit that in some sense looks far ahead and gives relatively heavy weight to the longer-term future will alter its permanent income less in response to unanticipated changes in circumstances that affect mainly current receipts than a unit that is short-sighted and will alter its permanent income more in response to changes that affect mainly distant receipts.

The mathematical model I suggested [3], for converting the rather commonplace observation that consumer units adjust consumption to longer-term income prospects into an operationally meaningful theory capable of being contradicted by observation, has two major elements and one important supplement. The elements are a model of income structure and a relation between consumption and income derived from the pure theory of consumer behavior. The supplement is an expectations model applied to aggregate time-series data in order to estimate permanent income. In the model of income structure, the concept of “horizon” is given a precise meaning that is highly relevant to the effect of an unanticipated change in circumstances: the “horizon” is a period of time used to dichotomize factors affecting income into “transitory” factors, all of whose effects on income are over within this time period, and “permanent” factors, whose effects last beyond this time period.¹ In the discussion of the pure theory of consumer behavior, the word “horizon” is used in a very different sense and one that turns out not to be relevant to the effect of an unanticipated change in circumstances: the “horizon” refers to the length of the period being analyzed; it betokens an end point, not a dividing line between shorter and longer periods [3, pp. 11, 14]. Moreover, no concept is singled out in the theoretical discussion as playing a role corresponding to that of “horizon” in the model of income structure. In the supplementary expectations model, one parameter is said to be related to the income structure “horizon.” However, the relationship is not made explicit, and I now believe that the one implied is wrong [3, p. 150].

This initial confusion of terms is not rectified in the rest of the book. The term is mostly used in the income structure sense: several pieces of empirical evidence are assembled suggesting that the “horizon” in that sense is approximately three years for consumer units in the United States for consumption as a whole, and the concept is used to predict the effect on consumption of unanticipated changes in circumstances. I have always been dissatisfied with these attempts to bridge the gap between the income structure model and the pure theory of consumer behavior. They are vague and indefinite in language, which simply reflected the vagueness and indefiniteness in my own understanding of the concept. And I am now persuaded that they led me into error and have misled others as well. I may not have found even now a fully satisfactory
economic interpretation of the mathematical construct of the horizon, but I believe I can give a better and less ambiguous formulation than that contained in my book.

This paper presents the improved formulation. The occasion for my rethinking the issues and writing this paper is twofold: (1) In an interchange on the permanent-income hypothesis, Robert Eisner and Hendrik Houthakker exchanged comments that exhibit particularly clearly the misleading character of my earlier formulation.² (2) In a paper on “Windfall Income and Consumption” prepared for a conference on consumption at the Wharton School, Ronald Bodkin presented the results of a test of the permanent-income hypothesis along the lines of one that I suggested in my book. In this connection he was troubled by what seemed to him, and rightly so, ambiguities in my discussion of how to determine the effects of windfall receipts on consumption [1].

The Eisner-Houthakker interchange and the Bodkin paper raise three sets of issues with which this paper deals: (1) the economic meaning of “horizon” and its use to predict the effect of unanticipated changes in circumstances—the particular change dealt with by both is a windfall in the form of an unanticipated cash receipt; (2) the effect of windfalls, and more generally any unanticipated change, on permanent and transitory components of income; (3) the effect of windfalls on the lack of correlation between transitory and permanent components of income that is assumed in the permanent-income hypothesis. In addition, the paper considers (4) the relation between the supplementary model applied to time-series data and the horizon.

1. The Meaning of Horizon

It is easier to say what the horizon is not than what it is. My estimate that the horizon is about three years does not mean that permanent income is to be defined as a three-year moving average of measured income. This is perhaps the most egregious error that has been made in interpreting the permanent-income hypothesis and the concept of horizon, and one for which my own loose use of the term deserves much blame despite my explicit caveats against such an interpretation [3, pp. 23, 213]. The error has two components. First, permanent income is of the nature of the mean of a hypothetical probability distribution, not the mean of a sample, so it cannot be calculated from actually experienced measured incomes, although these may of course be used to construct an estimate of permanent income (see Section 4). Second, a three-year horizon does not mean that consumers plan for only three years and seek to balance their books over that interval. On the contrary, it requires that consumers look beyond three years. It is of the nature of a mean length of time, not of total duration.

Economically, “permanent income” is significant as an index of wealth, where wealth is to be regarded as including not only the nonhuman assets we ordinarily designate by that term but also human capacities. The permanent-income hypothesis regards the consumer unit as having at any moment of time some estimate of his wealth, presumably in the form not of a single value but of a probability distribution of alternative values, and as adjusting his flow of consumption to an estimate of wealth formed from the probability distribution (in the simplest case, its expected value). If \( W \) stands for this estimate of wealth, then the permanent-income hypothesis in its most general form asserts that

\[ c_p = f(W) \]

(1)
where \( c_p \) is permanent consumption or “planned” consumption expressed as a rate of flow per some unit of time, and the function \( f(W) \) may of course include variables other than \( W \). The specific variant of the hypothesis that I have proposed gives a particularly simple form to the function \( f(W) \). It asserts that

\[
c_p = qW,
\]

(2)

where \( q \) is a number that is independent of \( W \)—though it does, of course, depend on other variables (such as market rates of interest, the fraction of \( W \) consisting of nonhuman assets, and so on).

On this level, it is clear conceptually how to handle some windfalls, but not others. A windfall is an unanticipated favorable change in circumstances, which is to say, a change that promises additional receipts not previously allowed for by the consumer unit in his estimate of \( W \). These additional receipts may be wholly coincident in time with the occurrence of the windfall, as, for example, when an individual unexpectedly finds on the street a packet with no identification containing $1000 which he gets to keep because no valid claim is made for it. Such a windfall raises \( W \) dollar-for-dollar. The $1000 the individual finds is indistinguishable from the rest of his wealth, except that it is of a particular form (nonhuman wealth), and so may change the fraction of his total wealth that is of that general form, even after he has made the feasible alterations desired by him in its specific form. The change in form of wealth may in turn change \( q \). This qualification aside, the consumer unit can be expected to increase consumption by \( q \) times $1000.

The additional receipts need not, however, be coincident with the occurrence of a windfall. For example, an individual may receive word that he has come into a wholly unanticipated inheritance to be paid to him at some specified future point in time, or he may learn to his surprise that he has been named a beneficiary in the will of a living person, so that the receipt will be at some indefinite future time, or he may find himself possessed of hitherto unsuspected talents that will enable him to earn larger sums in the future than he had allowed for in estimating \( W \). Such windfalls raise \( W \) by an amount that depends not only on the dollar value of the additional future receipts but also on their dating and on how the individual forms his estimate of \( W \) from his estimated future receipts.

To analyze such windfalls, we must go beyond the formulation given in (2). Moreover, (2) is not very useful for the interpretation of available data. These data do not give estimates of \( W \). They typically tell something about receipts and payments, little about the value of nonhuman assets, and even less about the value of human capacities. To convert (2) into a more useful form, we may proceed, as I did in my book, to replace \( q \) by the product of two factors: one expressing the permanent income flow as a fraction of the stock of wealth, the other expressing consumption as a fraction of income. Symbolically, let

\[
q = kr,
\]

(3)

so that

\[
c_p = krW = ky_p,
\]

(4)

where \( k \) is the average propensity to consume, \( r \) is an undefined term having the dimensions of an interest rate, and \( y_p = rW = \) permanent income.
If we require that \( rW \) on the average (over people or dates) approximate in size the magnitude we ordinarily term “income,” the observed average propensity to consume will give an estimate of \( k \) (with the usual qualifications about group transitory effects), and the problem reduces to estimating the size of \( r \). The size of \( r \) is in turn related to the estimation of \( W \) in view of the requirement that \( rW \) approximate average measured income. It is at this stage that the concept of horizon enters.

To simplify matters, consider first consumer units that own no nonhuman assets except some balancing resource, which might be the possibility of borrowing, to carry them from one pay period to the next, and expect all of their future receipts other than the yield from this resource to be from human capacities, these capacities themselves being regarded as homogeneous. We can then regard \( W \) as the discounted value of anticipated future receipts plus the amount of the balancing resource, where the discount rate is a “subjective” rate that need bear little or no relation to any market rate of interest, since human capacities cannot be bought or sold, though they can of course be improved through training. A shortsighted unit (equivalent to short horizon) will weight receipts in the near future more heavily compared to distant receipts than a long-sighted unit (equivalent to long horizon). We can interpret this as meaning that the former uses a higher discount rate than the latter in converting future receipts into present wealth. Let \( r \) be this subjective discount rate. The horizon can then be defined as \( 1/r \), or “the number of years purchase” implied by the discount rate. On this definition, my estimate that the horizon is on the average something like three years implies a subjective discount rate of 0.333. A unit with a shorter horizon would use a still higher discount rate, a unit with a longer horizon, a lower discount rate. For simplicity, I speak—and shall continue to do so—of a single discount rate as applicable to all future receipts, regardless of their timing. A completely general analysis would allow for the possibility that the discount rate may be a function of the time that elapses until the receipt occurs.

To avoid misunderstanding, it is perhaps worth noting explicitly that the level of the discount rate does not imply anything about whether wealth is being consumed, maintained, or increased. That is determined by \( k \). If \( k \) is above unity, wealth is being consumed; if \( k \) is equal to unity, being maintained; and if \( k \) is less than unity, being increased. The use of different discount rates has offsetting effects on \( y_p \). The higher the value used for \( r \), the lower will be the value assigned to \( W \), and conversely. Whether the product \( rW \) is higher or lower depends on the time shape of expected future receipts. It is perhaps worth noting explicitly also that the relevant magnitudes are expected “real” receipts, not nominal money receipts, and hence that \( r \) is to be interpreted as a “real” discount rate. To avoid complications introduced by expectations of changing price levels, we shall suppose that the price level is expected to remain constant, or what is equivalent, that all magnitudes are adjusted for expected changes in prices.

To state these relations more formally, let \( R(\tau) \) be anticipated receipts (which may be positive or negative) at time \( \tau \). Since these receipts occur at particular points in time—when the pay envelope, or the dividend check, or the rent is received—\( R(\tau) \) is zero almost everywhere, nonzero at only a denumerable number of points in time. Let \( r(T) \) be the discount rate applicable at time \( T \), assuming continuous discounting, and let \( D(T) \) be the accumulated balancing resource (depreciation reserve if positive or net borrowing if negative) that enables the consumer unit to
bridge the gap from one receipt to the next or to adjust for variations in the size of successive receipts. Then wealth at time $T$ is given by

$$W(T) = \sum_{(\tau_i)} e^{-r(\tau_i-T)} R(\tau) + D(T), \quad (5)$$

where $(\tau_i)$ stands for the set of values of $\tau$ which are greater than $T$ and for which $R(T) \neq 0$. For simplicity, assume that $D(T)$ yields interest continuously at a rate of $r(T)$ if positive or requires the payment of interest at the rate of $r(T)$ if negative. Permanent income is then given by

$$y_p(T) = r(T)W(T). \quad (6)$$

Note that expressing the rate of discount as a function of the date for which the wealth estimate is constructed does not conflict with the assumption that the rate of discount is the same whatever the elapsed time until a receipt of a particular category occurs. At any point in time $T$ a single rate is used for all future receipts whatever may be the elapsed time $(\tau - T)$, but this single rate can vary with $T$.

If all anticipations were realized and if $r$ were to remain constant, then—in the absence of consumption expenditures—wealth would grow at a rate equal to permanent income. If in addition $k = 1$, so that consumption equals permanent income, then wealth would remain constant. True, over time intervals for which $R(T)$ is zero, the financing of consumption involves a drain upon depreciation reserves or borrowing. But this is just balanced by the increase in the present value of subsequent receipts as a result of their coming closer in time.\(^5\) Similarly, at points where $R(T) \neq 0$, the first term on the right-hand side of (5) takes a sudden drop as a receipt occurs, but this is just matched by the addition of the whole of the receipt to $D(T)$ to replenish the depreciation reserve or to repay borrowing, so again wealth remains intact.\(^6\)

The situation is complicated if the assets are of more than one type, such as different kinds of human capacities, or different kinds of nonhuman wealth, or both human and nonhuman wealth. Suppose, temporarily, that the markets for the different assets are perfect, in the sense that the assets can be exchanged for one another at prices that are the same regardless of the direction of exchange. There would then be one subjective rate for each consumer unit for all assets, since if subjective rates differed, the consumer unit would have an incentive to shift the composition of its assets away from those with low subjective yield to those with high subjective yield. This subjective rate cannot, however, be identified in principle with any market rate and need not be the same for different consumer units. Different types of assets could have different market rates, so there would be no such thing as “a” market rate, whereas there is “a” subjective rate for each consumer unit. The reconciliation is that the subjective rate includes not only pecuniary returns but also nonpecuniary returns. Under the circumstances envisaged, each consumer unit would adjust the structure of its assets so that these nonpecuniary returns differed by just enough to equalize total returns. For example, currency generally yields a zero market rate of interest. If the subjective rate is $r$ per year and a consumer unit holds currency, this implies that it values the services rendered by currency as worth $r$ cents per year per dollar held. To keep the books straight, the value of services rendered by currency (and similarly the difference between $r$ times the values attached to other assets and the money yield on them) must in principle be included in consumption.\(^7\)

Although the value of $r$ cannot in principle be identified with any particular market rate of interest, its level for any consumer unit will be affected by and related to the level and structure
of market rates. Moreover, in practice, there might be some asset or assets which typically yielded little or no nonpecuniary returns to most consumer units, and whose yield might therefore serve as an empirical estimate of $r$.

The typical situation is intermediate between the two just considered. Some assets, particularly human capacities, are not directly bought and sold at all. They can be exchanged for other assets only to a limited extent or at a temporal rate fixed by conditions largely outside the control of the individual consumer unit. Other assets, such as currency, deposits, and government bonds are traded in essentially perfect markets. Still others, such as small owner-operated businesses, are traded in highly imperfect markets involving wide differences between buying and selling prices. Still others, such as future earning capacity, can be converted into property only through transactions (e.g., personal loans) involving large differences between rates of interest paid and received, equivalent to a wide difference in the buying and selling price of an asset. Under these circumstances, there may be different subjective rates for different assets, though it remains true that none need be identified with any particular market rate.\footnote{The general rate $r$ is a weighted average of these specific rates, its precise level determined by the rate used for each type of asset and the relative importance of the different assets. The horizon $1/r$ is likewise a weighted horizon. The existence of several specific rates is an additional reason why the average rate, $r(T)$, might vary over time. Even though each rate stays the same, the weights may change.}

It is perhaps worth reiterating that the relevant horizon need not be identical for all consumer units or even for one consumer unit for different categories of consumption. For example, it seems plausible that consumer units who receive their income primarily from human capital may have a different, presumably shorter, horizon than units who receive their income from property.\footnote{Similarly, it seems plausible that a unit may adopt one horizon for determining housing expenditures and another for food expenditures because the types of expenditures involve different time structures of future commitments. The use of different horizons by a single consumer unit for different types of expenditures is equivalent to the use of different discount rates for different categories of assets. Presumably, neither situation would arise if all assets could be exchanged in perfect markets.}

In terms of this conceptual framework, it is clear how to handle all windfalls that take the form of anticipated additional receipts at particular dates.\footnote{In terms of this conceptual framework, it is clear how to handle all windfalls that take the form of anticipated additional receipts at particular dates. The consumer unit discounts the additional receipts to the present at a rate that may depend on the form of the additions to future receipts. The result is added to $W$. In addition, the rate $r$ may be revised to allow for the different composition of assets. The product of the revised rate and the revised estimate of wealth is the revised permanent income. The average propensity to consume, $k$, may also be revised to allow for the different composition of assets. The product of the revised $k$ and the revised permanent income is the new level of consumption. If, for simplicity and because it will often be plausible to do so, we neglect any revisions in $r$ and $k$, the increment to permanent income as a result of the windfall is $r$ times the present value of the additional receipts, and the increment to consumption is $k$ times the increment to permanent income.}

2. Windfalls and Transitory Income
According to the view just expressed, a windfall raises both wealth and permanent income, the increment in wealth being indeed a measure of the size of the windfall. What effect does the windfall have on the transitory component of measured income? This question has been raised and discussed explicitly primarily for windfalls for which all of the additional receipts come at the same time as the windfall itself. It is tempting to say that such a windfall raises transitory income by the excess of the windfall over the increase in permanent income, that the windfall is, as it were, to be regarded as divided into two parts, an increase in permanent income and an increase in transitory income, the division depending on the horizon, so that if the horizon is three years, one-third of the windfall is permanent income and two-thirds is transitory income. It is but another small step to regard the windfall as distributed over three years, one-third becoming permanent income in each year. The initial step therefore easily leads to the erroneous concept of permanent income as a moving average of measured income.

I fell into this trap, at least to the extent of the first step, and Eisner followed me.\textsuperscript{11} The stated conclusions are wrong, deriving indirectly from a confusion of stocks and flows. Though a cash windfall raises permanent income, it can nonetheless also be true, paradoxical though it may seem, that the whole of the windfall is a transitory receipt, and raises the transitory component of income by the time rate of flow that corresponds to the whole of the windfall. And windfalls that give rise to future receipts can give rise to negative transitory components of income.

The transitory component of income ($y_t$) is defined as the difference between measured income and permanent income. Measured income, in turn, corresponds to the concept labeled “income” in budget studies and other statistical records. Its precise definition varies from one study to another. Always defined for a particular accounting period, usually a year, it is equal to the sum of specified (positive or negative) cash and imputed receipts during that period with occasional adjustments that are designed to go part way from a “cash” to an “accrual” concept. For present purposes, we may regard the items to be included in measured income as corresponding to those included in $R(\tau)$, the series of receipts from which wealth is estimated, plus, for logical completeness, the (positive or negative) receipts from interest on what we have called the depreciation reserve, $D(T)$.

Although the receipts to be included in measured income may correspond to those included in $R(\tau)$, we cannot take $R(\tau)$ itself as defining measured income. The reason is that we want measured income to be a rate of flow, to have the dimensions of dollars per unit time, like permanent income or permanent consumption, so that we can write

$$y_m(T) = y_p(T) + y_t(T),$$

(7)

where $y_m(T)$ and $y_t(T)$ are measured and transitory income at time $T$.

However, $R(\tau)$ is not a flow; its units are simply dollars, like $W(T)$, which is a stock at a point of time. One formal solution would be to define $y_m(T)$ as the time rate of receipt corresponding to the payments $R(\tau)$,\textsuperscript{12} but this is operationally useless since it would yield an infinite time rate of receipt at the points in time when $R(\tau) \neq 0$, and a zero rate elsewhere.\textsuperscript{13}

I can see no way to get a meaningful concept of measured income to fit into an equation like (7) except by explicitly introducing an accounting period and defining measured income in terms of receipts during that period. We can still express measured income as a time rate of flow at a
point in time, like permanent income, but the rate of flow will have to be computed from receipts during a specified period rather than being the limit of the ratio of receipts during a period to the length of the period as the length of the period approaches zero. Since the accounting period could, of course, be chronologically short, the difference will be of no practical importance for many purposes and it may appear pure pedantry for me to insist on the difference, yet I believe it is conceptually basic for the present problem.

One possible definition along these lines of measured income at a point in time is the annual rate of flow corresponding to the week (or month, or year, or decade) of which that point in time is the center. Formally, let \( A \) be the length of the accounting period. Define measured income at time \( T \) for an accounting period \( A \) as

\[
y_m(T, A) = \frac{1}{A} \sum_{(\tau_a)} R(\tau) + \frac{1}{A} \int_{T-A/2}^{T+A/2} r(T') D(T') dT'.
\]

(8)

where \((\tau_a)\) is the set of values of \( \tau \) for which \( T - A/2 \leq \tau < T + A/2 \) and for which \( R(\tau) \neq 0 \), and \( T' \) is simply a variable of integration. The final term is included for logical completeness and consistency with our previous treatment. Except for the final term, measured income so defined is of course a step function, having the same value over any time period for which the shifting boundaries of the accounting period neither drop out nor add a nonzero receipt. For example, for \( A \) equal to one month and a unit whose only receipt is a paycheck at the end of each month, \( y_m \) would be constant from the midpoint of one month to the midpoint of the next except for interest received or paid on the depreciation reserve.

Transitory income at time \( T \) could now be defined simply by substituting (8) and (6) into (7). But if this were done, it would implicitly involve different time references for \( y_m \) and \( y_p \); \( y_m \) as defined for time period \( T \) is an average over the interval \( T - A/2 \) to \( T + A/2 \), whereas \( y_p \) is an instantaneous rate at time point \( T \). To make the time reference of the several terms the same, we can replace \( y_p(T) \) by its average value over the same time interval. Designate this average by \( \bar{y}_p(T, A) \), the second argument denoting the length of the accounting period, and introduce a second argument similarly into \( y_r \). This gives

\[
y_r(T, A) = y_m(T, A) - \bar{y}_p(T, A)
\]

(9)

\[
= \frac{1}{A} \sum_{(\tau_a)} R(\tau) + \frac{1}{A} \int_{T-A/2}^{T+A/2} r(T') D(T') dT' \\
- \frac{1}{A} \int_{T-A/2}^{T+A/2} r(T') W(T') dT'.
\]

For simplicity, assume that calculations are made retrospectively as of \( T + A/2 \), so that actual depreciation reserves and those that enter into the calculation of \( W \) are the same over the interval \( T - A/2 \) to \( T + A/2 \), and assume further that \( r \) is constant over the same interval. If we now replace \( W(T') \) by its value as given by (5), the terms in (9) involving the depreciation reserve will cancel, leaving

\[
y_r(T, A) = \frac{1}{A} \sum_{(\tau_a)} R(\tau) - \frac{r(T)}{A} \int_{T-A/2}^{T+A/2} \sum_{(\tau)} e^{-r(T)(\tau-T)} R(\tau) dT'.
\]

(10)

Transitory income so defined is precisely the average rate at which depreciation reserves must be added to or subtracted from during the accounting period over and above interest on the reserve in order to keep wealth intact. Note that because we assume the calculation to be made
retrospectively at the end of the accounting period, the elements of $R(\tau)$ that enter into the first term of the right-hand side of (10), are the same as those for the corresponding period which enter into the second term; in addition, anticipated receipts after the accounting period enter into the second term. Without this assumption, we cannot use the same symbol $R(\tau)$ in both terms. We would have to distinguish actual from anticipated receipts.

Let us consider now the effect of a windfall that occurs during the accounting period. Suppose the windfall consists of the discovery that a cash receipt of $U$ (for unanticipated) dollars will occur at time $T_0$, and let $R^*(\tau)$ be the new stream of receipts, so that

$$R^*(\tau) = \begin{cases} R(T_0) + U & \text{for } \tau = T_0, \\ R(\tau) & \text{for } \tau \neq T_0. \end{cases} \quad (11)$$

Similarly, let an asterisk attached to any other variable convert it to its new value after the windfall.

2.1. $T_0$ Is Within Accounting Period. Suppose first that not only the windfall but also the cash receipt occurs between $T - A/2$ and $T + A/2$. Then

$$Y^*_t(T, A) - y_t(T, A) = \frac{U}{A} e^{-rA}, \quad (12)$$

where $f$ is the fraction of the accounting period elapsed before receipt of the windfall and where, to simplify the expression, I have written $r$ for $r(T)$.\textsuperscript{16}

If the windfall is received at the very beginning of the accounting period, so that $f = 0$, then transitory income is raised by $U/A$, which is to say by the whole of the windfall converted into an annual rate by treating it as received during an accounting period. For example, if time is expressed in years, $A$ is a month, and the windfall is the finding of $1000, the transitory component during the month subsequent to the windfall is at the annual rate of $12,000. If $A$ equals a year, we can say that the transitory component is equal to the whole of the windfall. But this is really an erroneous and inaccurate statement because the transitory component is a rate of flow and has the units of dollars per unit of time, while the windfall is an amount and has the units simply of dollars; the numbers come out the same because of the accident that $A$ happens to be unity; but the equality is not invariant to an arbitrary change in the time dimension; the statement is analogous to one asserting that the rate of speed of a car equals the distance covered.

To continue for a moment with the special case $f = 0$, wealth at the outset of the accounting period is raised by $U$, the amount of the windfall, and hence permanent income is raised by $rU$. This is the amount by which the rate of consumption can be increased while keeping wealth intact. On our assumption that the depreciation reserve to which $U$ is added in the first instance has a continuous yield at the rate of $r$, the interest on the addition to the depreciation reserve is just enough to pay for the additional consumption, hence the transitory component is—as noted earlier—the average rate at which the depreciation reserve is added to from receipts during the accounting period to keep wealth intact.\textsuperscript{17}

These comments may help to explain why transitory income rises by less than $U/A$ when the windfall comes after the beginning of the accounting period. Our assumption that wealth is calculated throughout the accounting period as if the unit foresaw the events of that period (i.e., the assumption that actual and expected receipts are equal during the accounting period) means

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that a windfall anytime during the period raises wealth at the outset of the period by its
discounted value or by \( U \cdot e^{-rA} \). But this means that permanent income is also raised at the outset
of the period. Now, however, there is for a time no interest on a raised depreciation reserve to
finance the higher level of spending possible while keeping wealth intact; it would have to be
financed by drawing down the depreciation reserve. Part of the windfall would have to be used,
as it were, to make good this drain, leaving only the rest as a net addition to depreciation
reserves. Hence transitory income is smaller when \( f > 0 \) than when \( f = 0 \).18
If we suppose \( k = 1 \), which is to say consumption equal to permanent income, then wealth is in
fact maintained intact and we can readily write down the effect of the windfall on wealth,
permanent income, and measured income:19

\[
\begin{align*}
\bar{W}^* (T, A) - \bar{W}(T, A) &= Ue^{-rA}, \quad (13) \\
\bar{y}_p^* (T, A) - \bar{y}_p(T, A) &= ru^{-rA}, \quad (14) \\
\bar{y}_m^* (T, A) - \bar{y}_m(T, A) &= Ue^{-rA} \left( \frac{1}{A} + r \right). \quad (15)
\end{align*}
\]

If \( k < 1 \), there will be, through saving, a greater increase in wealth during the accounting period
than otherwise, so the increase in average wealth will be greater than that shown by (13).
Permanent income and measured income will likewise rise by more than they would have
otherwise by virtue of the yield on the greater wealth. If \( k > 1 \), so that there is dissaving,
the converse of each of these statements holds. But on our assumption that the depreciation reserve
yields income at a rate of \( r \), none of these effects will alter the increase in transitory income. It is
given by (12) whatever the value of \( k \).

2.2. \( T_0 \) Is Subsequent to the Accounting Period. Suppose that although the windfall occurs
during the accounting period, the associated cash receipt does not occur until some date after the
end of the accounting period. Then, from (10, and (11),

\[
y_* (T, A) - y_1 (T, A) = \frac{-U}{A} e^{-rL} (e^{rA} - 1). \quad (16)
\]

where \( L \) is the length of time between the beginning of the accounting period and the cash receipt
of \( U \).20 The increment to transitory income is negative because the windfall raises wealth, and
therefore also permanent income, but provides no additional receipts to enter into measured
income. Why this magnitude? \( U e^{-rL} \) is the discounted value of the unanticipated receipt and is
therefore the increment to wealth at the beginning of the accounting period as a result of the
windfall; i.e., it is the size of the windfall. Then \( r U e^{-rL} \) is the amount by which permanent
income is raised. This is the additional rate at which the depreciation reserve can be depleted
with wealth kept intact. To a first approximation, therefore, this is the negative transitory
component as a result of the windfall, as can be seen by replacing \( e^{rA} \) with the first two terms of
its Taylor’s expansion around \( rA = 0 \), that is, \((1 + rA)\), in which case the right-hand side of (16)
reduces to simply \(-rUe^{-rL}\). The actual transitory component is larger in absolute value because it
allows in addition for the decline of interest receipts on the depreciation reserve as a result of the
larger drain on it that would be required to finance spending equal to the higher permanent
income. That is, not only does permanent income rise but also measured income declines.
Needless to say, the shorter the accounting period, the less the latter effect, and the closer the fall
into transitory income approaches to the rise in permanent income.
If we suppose that $k = 1$, the increments to wealth, permanent income, and measured income for this case are

$$\bar{W}^*(T,A) - \bar{W}(T,A) = Ue^{-rL},$$  \hfill (17)

$$\bar{y}^*_p(T,A) - y_p(T,A) = rUe^{-rL},$$  \hfill (18)

$$\bar{y}^*_m(T,A) - \bar{y}_m(T,A) = -\frac{U}{A} e^{-rL} (e^{rA} - 1 - rA).$$  \hfill (19)

### 2.3. Effects of Restrictive Assumptions.

The specific assumptions made above have enabled us to get rather simple and definite answers but, so far as I can see, have not distorted the answers. If the calculations were not made retrospectively as of the end of the accounting period, it would be necessary to introduce a distinction between actual and anticipated receipts and actual and anticipated depreciation reserves, and this would introduce other possible transitory components and effects on permanent and measured income; but the ones we have isolated would still be there. Similarly, if the depreciation reserve did not earn interest continuously at the rate used in discounting future receipts, it would be necessary to examine explicitly how an unanticipated receipt during the accounting period was invested and how this affected measured income, and it might then be that the form which the windfall initially takes is important. Similarly, it would be necessary to examine the form of additional drains on the depreciation reserves as a result of unanticipated receipts after the accounting period.

Finally, we have used a single discount rate and treated it both as constant during the accounting period and, more important, as unaffected by the windfall. In practice, as we have seen, not all assets need yield the same subjective rate, because of imperfections in the capital market. If a windfall adds to or subtracts from a set of assets that have a relatively high or a low subjective rate, the single weighted-average rate $r$ must be adjusted accordingly. Like the other departures from our assumptions, this would require modifications in the wording of the preceding analysis without any substantial change in the conclusions.

These conclusions would remain: (1) transitory income as a rate of flow cannot be defined satisfactorily without specifying an accounting period; (2) a windfall that produces additional receipts within the accounting period in which it occurs will raise transitory income, permanent income, and measured income for that period; (3) the amount of the rise in transitory income depends not only on the magnitude of the additional receipts but also on when they occur and on the length of the accounting period; (4) a windfall that produces additional receipts after the accounting period in which it occurs will lower transitory income and measured income but raise permanent income in that period and subsequent periods up to the one in which the receipt occurs.

### 3. The Correlation Between Permanent and Transitory Components of Income

Concentration of attention on cash windfalls that may be expected to raise both permanent and transitory components has led to questioning of the assumption that there is a zero correlation between transitory and permanent components of income; an assumption that plays an important role in the permanent-income hypothesis as I have formulated it. The preceding analysis makes it clear that the questions raised have reflected mainly the incomplete range of windfalls considered. As we have seen, all windfalls raise permanent income. Some raise transitory
income, some lower it; when they lower transitory income, they may do so for a number of successive accounting periods. It is possible that windfalls introduce a positive correlation between transitory and permanent components of income; it is possible also that they introduce a negative correlation. I see no way to establish a general presumption one way or the other; the result depends on the distribution of windfalls by kind and on the length of the accounting period. In consequence, I see no reason for revising, on the basis of considerations of this kind, the tentative assumption of zero correlation that is embodied in my formulation of the permanent-income hypothesis. And, of course, similar statements are valid for unanticipated unfavorable changes in circumstances.\textsuperscript{24}

It may be, of course, that in a particular problem dealing with specific windfalls affecting a specific group, a presumption can be established that the correlation is in a particular direction and significant in magnitude. For any such problem, the analysis will have to be complicated to allow for the correlation, whose magnitude can probably be inferred from the circumstances that require taking account of it.

More generally, the offsetting effects of windfalls on the correlation between permanent and transitory components of income are further diffused by the existence of transitory components of income of other kinds. As we have seen, for a particular consumer unit and a particular time period, a transitory component may be fully anticipated. When we speak of the correlation between such transitory components and permanent components, we implicitly have in mind either a number of such time periods or a number of consumer units, whereas with windfalls (and their opposite) we can have in mind a single unit and the probability distribution of receipts in a single subsequent time period. We shall not often be misled if we do not specify precisely which of these we have in mind; sometimes, however, as in problems like those mentioned in the preceding paragraph, it is important to consider the situation more explicitly. “Randomness” or “chance,” ideas which underly the concept of “zero correlation,” are sophisticated concepts. What is “random” to me may be well determined in terms of the information available to someone else; and conversely, what is well determined to me may appear “random” when I am considered as one of a set of entities by an outside observer. The concept of “transitory income” is formally definable for a single consumer unit in terms of its anticipations, as in (6). Its interpretation in empirical work depends on particular circumstances and the particular group of time units or consumer units considered.

4. The Estimation of Permanent Income

In connection with the analysis of aggregate data, I have used an estimate of permanent income constructed from past data as follows:

\[
E[y_p(T)] = \beta \int_{-\infty}^{T} e^{(\beta-x)(T-T')} y_m(T')dT',
\]

where \(E\) denotes “estimate of”; this notation has been slightly altered from that in my book to conform with that used above [3, p. 144]. This estimate has two logically distinguishable elements: (1) an exponentially weighted average of past measured incomes adjusted for trend, the weights being given by \(\beta e^{(T-T')}\), where \(T\) is the date for which the estimate is constructed and \(T'\) covers the whole range of earlier dates so that the weights decline as one goes farther back; (2) an adjustment for trend at the continuous rate of \(\alpha\) per unit of time.
Although I used this estimate in my book only for aggregate data, I shall discuss it here as if it applied to an individual consumer unit, since my present purpose is to see how the value of $\beta$ can be connected with the concept of horizon discussed above, and that concept is defined strictly for a single consumer unit.

4.1. Weighted Average of Past Measured Incomes. The rationalization that I gave in my book for using a weighted average of past incomes was in terms of the kind of expectations or adjustment model that has become familiar through the work of Cagan, Nerlove, Koyck, and others. In this model, $\beta$ measures the speed of adjustment of expected magnitudes to actual magnitudes.

This rationalization requires the interpretation of $y_p(T)$ as “the ‘expected’ or predicted value of current measured income.” [3, p. 143]. This is more satisfactory for the aggregate or average permanent income of all consumer units than it would be for an individual consumer unit, since—once allowance is made for secular growth—it is plausible that deviations between expected measured income and expected permanent income, which may well be sizable for individual units, largely or wholly cancel for all units so that one can be taken as an estimate of the other. However, even for all consumer units it is not fully satisfactory. For example, during the great depression, say from 1931 to 1934, it is not implausible that consumer units on the average anticipated that their measured incomes would be lower than their permanent incomes and conversely during booms, so that the figure I have interpreted as equal to expected permanent income cannot also validly be interpreted as expected measured income.

Another difficulty, for our present purpose, with the rationalization that identifies expected measured with expected permanent income is that it gives no rigorous connection between $\beta$ and the horizon. In my book [3, p. 150], I asserted a connection but did not demonstrate one:

The value of $\beta$ turned out to be .4, implying an average lag of $2\frac{1}{2}$ years, or an “effective weighting period” of 5 years. In terms of our hypothesis, this period is presumably related to the horizon implicit in judgment of permanent income by individual consumer units. It seems plausible that this period would be longer for aggregate data than the corresponding horizon for individual units, due to the averaging out of random factors.

The final sentence does not seem unreasonable for an adjustment coefficient used to derive expected measured income but I do not now believe it to be correct for a horizon defined as in the earlier sections of this paper. Suppose all consumer units used the same value of $r$ in discounting future returns. Then that same value of $r$ would be the one to use in converting the aggregate of anticipated returns into aggregate wealth and aggregate permanent income. If the values of $r$ differ, then a rather complex weighted average is relevant, but I see no reason to assert that this weighted average has a systematic bias relative to individual values of $r$.

Similarly, the earlier part of the quotation, with its weasel word “presumably,” avoids the key issue: why should the horizon be identified with the “effective weighting period” rather than the average lag? I suspect that the reason I then thought so was because I was implicitly and erroneously thinking of the kind of horizon that I unfortunately used in my initial theoretical analysis, namely, a cutoff period.
An alternative rationalization of the exponential weighting in (20) that is relevant to individual data as well as to aggregate data and that avoids these difficulties is to regard individuals as taking their past experience, adjusted for trend, as the best single estimate of their likely future experience. Set $\alpha$ temporarily equal to zero, to eliminate the problem of a trend. Assume that measured income at time $T'$ in the past is an estimate of anticipated measured income $T - T'$ time units in the future, and set $\beta = r$. Equation (20) then differs from equation (6), which is our basic definition of permanent income, only in being a continuous integral rather than a summation over discrete points in time. And this difference is only expositional, since $y_m(T')$ is in practice a discrete series at annual intervals.\(^{25}\)

Allowance for trend is required on this interpretation because of differences between current wealth and wealth at the time the past observed measured incomes occurred, as a result either of intended savings or of accidental but known capital gains or losses. Since the unit knows about such differences in wealth, they provide a reason for expecting future experience to differ systematically from past experience and hence should be taken into account in estimating permanent income. We can allow for them by treating the past increase in wealth as if it had occurred at a steady percentage rate, which we designate $\alpha$, and as if it were of the same composition as prior wealth. On these assumptions, the estimate of current permanent income can be regarded as made in two steps. First, each past measured income is adjusted to the value it would have had if wealth then had been equal to its value now. On the assumptions just made, the adjusted value of $y_m(T')$ is $e^{\alpha(T'-T)} y_m(T')$. Second, these adjusted values are used as estimates of the future receipts to be expected, the discounted value of which constitutes present wealth.

On this interpretation, $\beta$ is a direct estimate of $r$, which, as it happens, yields an estimate of the horizon closer to that yielded by other evidence than did my earlier assertion that $2/\beta$ was an estimate of the horizon. As I have noted, $\beta$ turned out to be 0.4, which is certainly rather close to the value for $r$ of 0.33 that we have been using.

4.2. Allowance for Trend. On the interpretation just presented, $\alpha$ is an estimate of the known past rate of increase in wealth. Capital gains and losses may be expected to cancel among consumer units at any point in time as well as over time. Hence $\alpha$ can be approximated by the planned past rate of increase in wealth. If $k$ had been equal to unity throughout, so that consumption equaled permanent income, the value of $\alpha$ in (20) would be zero. If $k$ had been less than unity, the consumer unit would have been saving to raise permanent income, so $\alpha$ should be positive; the converse would have been true if $k$ had been more than unity. If we continue to use a single rate of discount, the rate of change in permanent income is given by

$$\alpha = r(l - k).$$ \hspace{1cm} (21)

For $r = \frac{1}{3}$ and $k = 0.9$ (the approximate value of $k$ for the United States as estimated in my book), $\alpha = 0.033$, whereas I used a value of $\alpha = 0.02$ in my analysis of aggregate data. This value of $\alpha$ was derived from the secular trend in estimated per capita real consumption. For several reasons, it is not clear that these two estimates of $\alpha$ are fully comparable.
(1) Savings as defined in computing the numerical value of $k$ include only savings in nonhuman form, whereas for present purposes savings should include all forms of addition to wealth. This has effects working in different directions. It leads to an overestimate of savings (underestimate of $k$) insofar as saving in nonhuman form is, for the individual consumer unit, simply a conversion of human to nonhuman wealth as the unit ages. It leads to an underestimate of savings insofar as the reverse process is at work: nonhuman capital is being converted into human capital via expenditures on education or training, or human capital is being created through savings out of current income. If the aggregate $\alpha$ is intended, as I believe it should be, to be an average of the values for individual consumer units, then the effect that dominates will depend on how consumer units judge the expenditures on raising and training children: they may and in large measure doubtless do regard such expenditures as consumption even though for the economy as a whole they constitute replacement or increase in human capital, in which case the first effect will almost surely dominate, and $k$ from empirical data will on this account be an underestimate of the relevant value.

(2) Since $r = \frac{1}{3}$ is intended to include nonpecuniary as well as pecuniary returns from wealth, these returns should also be included as income and consumption, whereas most are excluded in the empirical data from which $k = 0.9$ is computed. Since inclusion of these nonpecuniary returns adds equal absolute amounts to numerator and denominator, it tends to raise the ratio of consumption to income when this ratio is less than unity. Hence their exclusion makes the observed $k$ an underestimate of the relevant value.

(3) The exclusion of nonpecuniary returns may also affect the value of $\alpha$ estimated from the trend in per capita real consumption. If the nonpecuniary returns excluded from the estimates of consumption have grown relative to estimated consumption, as seems highly likely in view of the apparent growth relative to total wealth of consumer durable goods and liquid assets yielding low pecuniary returns, then the trend in per capita consumption underestimates the value of $\alpha$ relevant to (21).

Combining these effects, there seems to be no irreconcilable contradiction between a value of $\alpha$ of 0.033 computed directly from (21) using $r = \frac{1}{3}$ and $k = 0.9$ and the value of $\alpha$ of 0.02 derived from the observed trend in per capita real consumption. For example, adjustment of $k$ to 0.94 because of effects (1) and (2), would alone produce a value of $\alpha$ of 0.02; and adjustment of $\alpha$ to 0.025 because of effect (3), would imply that a value of $k = 0.0925$ would suffice to equate the two estimates.

5. Conclusion

This paper suggests that permanent income of a consumer unit be interpreted as the product of an interest rate $r$ and a stock of wealth $W$; that the stock of wealth be interpreted as the present value of anticipated future receipts from both human and nonhuman assets discounted back to the present at subjective rates of interest that need not be the same for different categories of receipts but whose average value is $r$; and that the horizon be defined as $1/r$, or “the number of years purchase” corresponding to the interest rate $r$. 

From The Collected Works of Milton Friedman, compiled and edited by Robert Leeson and Charles G. Palm.
The paper demonstrates that this interpretation permits a straightforward calculation of the effects on consumption to be expected on the permanent-income hypothesis from an unanticipated change in circumstances. It demonstrates further that the occurrence of such unanticipated changes provides no reason to question the zero correlation between permanent and transitory components assumed in the specific version of the permanent-income hypothesis I have proposed.

The paper also offers a new interpretation of the exponentially weighted average of past measured incomes that I have used as an empirical estimate of permanent income from aggregate data. It suggests that the exponent $\beta$ in the weighting pattern be taken as equal to $r$, the interest rate used in calculating wealth. This interpretation links the permanent-income hypothesis and this estimation procedure more closely and rigorously than I have done heretofore and for that reason alone is aesthetically preferable. In addition, it links the trend-adjustment parameter $\alpha$ in the weighting pattern with the parameters of the consumption theory, thereby providing another indirect piece of evidence on the value of $r$, and it makes for a higher degree of consilience in the empirical estimates of the horizon derived from time-series data and budget data.

Throughout the paper, I have tentatively accepted 3 years as an estimate of the average horizon for the United States, which means a subjective rate of discount of 0.33. Nothing in this paper gives any reason to suppose that this estimate is widely in error. If it seems drastically out of line with widely quoted market rates of interest, it should be kept in mind that these rates apply only to a very limited range of assets; that most future receipts whose discounted value constitutes wealth come from assets that cannot be readily bought and sold or for which buying and selling prices differ widely. It should be kept in mind also that the value of $r$ required by the theory includes not only pecuniary but also nonpecuniary returns from the assets. These nonpecuniary returns rationalize the simultaneous ownership by consumer units of assets that have widely varying rates of pecuniary yield.

**APPENDIX A**

**Derivation of Effect of Unanticipated Receipt During Accounting Period on Measured Income**

We shall derive equation (15) by evaluating items (a), (b), and (c) of footnote 18.

Item (a) is simply equal to $U$.

To get items (b) and (c), we need to know the difference between $D(T)$ and $D^*(T)$ at each date in order to integrate. At any date in the accounting period other than $T_0$,

$$\frac{d[D(T') - D^*(T')]}{dT'} = rUe^{-rfA} + r[D(T') - D^*(T')].$$  \tag{i}

where the first term on the right is the extra drain on the depreciation reserve to finance the increased consumption equal to increased permanent income and the second term is the drain because of lower interest received on the lower depreciation reserve. This is a differential equation whose solution is

$$D(T') - D^*(T') = Ke^{rT'} - Ue^{-rfA},$$  \tag{ii}

From The Collected Works of Milton Friedman, compiled and edited by Robert Leeson and Charles G. Palm.
where $K$ can be chosen to satisfy an initial condition. At $T - A/2$, $D^* = D$, so if we use this as an initial condition,

$$Ke^{r(T - A/2)} = Ue^{-rfA}, \quad \text{(iii)}$$

or

$$K = Ue^{-rfA-r(T - A/2)}, \quad \text{(iv)}$$

giving for the time interval $T - A/2$ to $T_0$,

$$D(T') - D^*(T') = Ue^{-rfA}[e^{r(T' - (T - A/2))} - 1]. \quad \text{(v)}$$

At $T_0$, since $T_0 - (T - A/2) = fA$, this gives

$$D(T_0) - D^*(T_0) = U(1 - e^{-rfA}). \quad \text{(vi)}$$

This is the value the depreciation reserve approaches before allowance is made for the receipt of $U$; i.e., its limit as the approach is made from earlier dates. The receipt of $U$ produces a jump of $U$ in $D^*$ at $T_0$, hence the limit as the approach is made from later dates is

$$D(T_0) - D^*(T_0) = U(1 - e^{-rfA}) - U = -Ue^{-rfA}, \quad \text{(vii)}$$

which is precisely equal in absolute value to the discounted value of the transitory receipt, validating some of the statements made earlier.

If this value is used for the initial value at time $T_0$, it gives a $K$ equal to zero, which means that, as stated earlier, for the time interval from $T_0$ to $T + A/2$,

$$D(T) - D^*(T) = -Ue^{-rfA}; \quad \text{(viii)}$$

that is, the depreciation reserve is higher by precisely the amount of the windfall.

Item (b) is equal to the interest on the negative of (viii) for the interval $T_0$ to $T + A/2$, which is

$$r(1 - f)AUe^{-rfA}. \quad \text{(ix)}$$

Item (c) is given by

$$\int_{T - A/2}^{T_0} r[D(T') - D^*(T')] = rUe^{-rfA} \int_{T - A/2}^{T_0} (e^{r(T' - (T - A/2))} - 1) dT'$$

$$= U(1 - e^{-rfA} - rfAe^{-rfA}). \quad \text{(x)}$$

Finally, (a) + (b) − (c) gives, as the increase in measured receipts,

$$U[1 + r(1 - f)Au^{-rfA} - 1 + e^{-rfA} + rfAe^{-rfA}] = Ue^{-rfA}(1 + rA). \quad \text{(xi)}$$

Converted into a rate of flow this gives

$$y^m_m(T, A) - y_m(T, A) = Ue^{-rfA}\left(\frac{1}{A} + r\right). \quad \text{(xii)}$$

which is equation (15).

**APPENDIX B**

**Derivation of Effect of Unanticipated Receipt After Accounting Period on Measured Income**

Equation (v) needs only slight modification to apply to the whole of the accounting period for this case. It is necessary to replace $Ue^{-rfA}$ which was the increment in wealth at the outset of the accounting period in the prior case, by $Ue^{-rT}$, which is the same thing in this case. If we make
this substitution, multiply by \( r \), and integrate, we have the loss of interest on the smaller
depreciation reserve, or

\[
rUe^{-rL} \int_{T-A/2}^{T+A/2} (e^{r(T'-T-A/2)} - 1) dT' = Ue^{-rA}(e^{rA} - 1 - Ar).
\]

This is the only item for this case affecting measured income. To get an annual rate, we must
divide by \( A \). The negative of the result is then the change in measured income as given in
equation (19).

References

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(1960).
48 (December 1958), 972–90.
[4] FRIEDMAN, MILTON, and SIMON KUZNETS, Income from Independent Professional
Univ. Press), 1945.

Notes

1 The model of income structure was taken over from an earlier analysis of professional incomes [3, pp. 21–25],
[4, pp. 325–28, 352–64].
2 See [2, esp. fn. 3] and [5, esp. fn. 2].
3 I am greatly indebted to Armen Alchian and William Meckling for calling my attention to the difficulties raised
by these footnotes. The reformulation that follows was largely stimulated by their comments and owes much to their
suggestions.
4 This note, which was first drafted in 1959, has benefited greatly also from comments on the initial draft by
Hendrik Houthakker and Arnold Zellner.
5 A precise statement would have to be more elaborate, particularly when \( W \) is regarded as the mean of a
probability distribution. The problem is suggested by a lottery ticket. In advance of the drawing, the ticket enters
into \( W \) at its expected value; after the drawing, \( W \) goes up by the excess of the prize over the expected value if the
individual wins a prize or goes down by the expected value if he loses. The notion of “windfall” is therefore
logically identical with that of positive “profit” in the uncertainty theory of profit.
6 It is worth recording that we do have some relevant data on human capacities—age, occupation, education,
etc. One interesting direction of research would be to try to estimate human wealth from such indexes thereof and to
use such estimates in analyzing consumption.
7 Proof: Suppose \( R(\tau) \) is zero and \( r \) a constant for \( \tau = T \) to \( \tau = T + \Delta T \), and no consumption takes place, so that
the depreciation reserve changes only by virtue of interest received on it if positive or paid on it if negative. By
definition,
\[ W(T + \Delta T) = \sum_{(\tau_2)} e^{-r[(T + \Delta T - \tau)]} R(\tau) + D(T + \Delta T) \]
\[ = e^{r\Delta T} \sum_{(\tau_1)} e^{-r(T - \tau)} R(\tau) + e^{r\Delta T} D(T) \]
\[ = e^{r\Delta T} W(T), \]

where \((\tau_2)\) is the set of values of \(\tau > T + \Delta T\) for which \(R(\tau) \neq 0\), since by assumption no terms are added to the sum by extending the range of summation from \((\tau_2)\) to \((\tau_1)\). Replace \(e^{r\Delta T}\) with the first two terms of its Taylor expansion around \(\Delta T = 0\). This gives

\[ W(T + \Delta T) = W(T) + rW(T) \cdot \Delta T, \]

an approximation that approaches an equality as \(\Delta T\) approaches zero. The second term on the right-hand side is precisely permanent income accrued during period \(\Delta T\). If it is wholly spent on consumption, wealth remains unchanged.

6 Whatever the rate of consumption so long as it is finite, the amount at a point in time is zero, which is why the whole of the receipt is available to add to \(D(T)\). That is, the value of any finite multiple of the \(rW(T) \Delta T\) of the preceding footnote approaches zero as \(\Delta T\) approaches zero.

7 This is essentially what is done in the national income statistics for bank deposits and similar items when banks are treated as “associations of individuals.” It is not done in consumer budget data, except to a limited extent for owner-occupied housing. It might be highly desirable to include a similar allowance for “imputed income” for a wider class of assets.

8 Note that there need not be different subjective rates. For example, the phenomenon of the individual who holds savings deposits yielding, say, 3 per cent and at the same time pays on an installment contract involving paying, say, 18 per cent for a loan, need not involve different subjective rates. Because of the liquidity value attached to the savings deposits, the subjective rate on the deposits may match the 18 per cent rate paid on the loan.

9 If this were so, it would imply a life cycle in permanent income even for a consumer unit for which \(k\) is unity throughout and for which anticipations are realized throughout (a concept that itself raises some difficult issues), since the ratio of human to nonhuman wealth would change steadily as human wealth was transformed into nonhuman wealth. Of course, a life cycle would also be produced—and this seems a clearly important factor whether the preceding is present or not—by a dependence between the value of \(k\) and the age of the consumer unit [3, pp. 23–25].

10 The qualification is to avoid explicit consideration of the problems raised by uncertainty and multi-valued anticipations. These problems, though certainly important, are not directly relevant to the issues considered in this paper, except when the existence of uncertainty or perhaps the degree of uncertainty is an important factor making for imperfection of asset markets and thereby affecting the horizon as defined above.

11 “The transitory component is only the excess of the windfall over this element of permanent income” [3, p. 29].

“For a man who wins $100 at the races should, with the three-year horizon that Friedman postulates and a modest rate of interest, reckon that his permanent income is up by about $35. It would be only the remaining $65 that would constitute transitory income. … However, this consideration of the nature of the income components does suggest a problem as to the consistency of Friedman’s definitions with his assumption of zero correlation of permanent and transitory components” [2].

12 That is, as the derivative of the step function obtained by cumulating successive receipts.

13 Or, if account is taken of the depreciation reserve, a positive or negative rate equal to \(rD(T)\).

14 A yield \(r\) of 33⅓ per cent per year implied by a horizon of three years may seem radically out of line with market yields and hence inconsistent not only with the general theoretical interpretation presented in this paper but even more with the supplementary assumption that the depreciation reserve yields a continuous yield at the rate \(r\). However, it should be kept in mind that we are treating the value of nonpecuniary returns from particular assets as an addition to permanent income, consumption, and measured income. For this reason, \(r\) is to be taken as the market yield only on assets yielding no nonpecuniary returns. Rates paid on installment contracts, small loans, etc., are not strikingly out of line with an estimated subjective rate of 33⅓ per cent. Note also that the depreciation reserve in the general case where there are many assets may be held in the form of assets such as durable consumer goods, the imputed value of whose services would then be included in permanent income, consumption, and measured income.

15 Note that two time units are involved: (1) the accounting period equal to \(A\); (2) the period in terms of which the rate of flow is expressed, implicit in the units in which \(T\) is expressed. For example, \(A\) could be a month and \(T\) could be expressed in years. Then \(y_m(T, A)\) would give the annual rate of monthly receipts. The first unit is
This page contains a mathematical derivation involving the relationship between receipts, interest, and windfall gains. The text is from a scholarly work, likely in the field of economics or finance, discussing the transformation of variables and the implications of a windfall gain on wealth and interest receipts. The derivation includes equations and proofs, indicating a detailed analysis of the economic effects of windfall gains.
“If a windfall gain were to cause an increase in permanent income, as well as an increase in transitory income, the two would be correlated, and this Friedman rules out. Eisner’s strictures are based on an economic interpretation of permanent income which, whatever its other merits, has little or nothing to do with the statistical assumptions in which the permanent income hypothesis should be expressed, according to Friedman’s own exposition” [5, p. 993n].

24 Is it our optimistic bias that accounts for there being no obvious antonym to “windfall”?

25 Compare (5), (6), and (8). The absence of an explicit term for the depreciation reserve in (20) also reflects its being a continuous integral. An explicit depreciation reserve is needed in (5) to bridge the gaps between the discontinuous receipts.

26 About the only general exception is nonpecuniary returns from owner-occupied housing and from food grown for own use.

10/1/12